Grey Kangaroo

Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

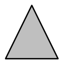
Enquiries about the Grey Kangaroo should be sent to:

challenges@ukmt.org.uk

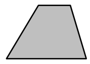
www.ukmt.org.uk

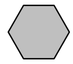
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	B	C	B	D	E	E	C	B	C	E	D	D	D	A	B	C	C	D	D	B	E	A	C

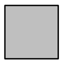
1. Which of the shapes below cannot be divided into two trapeziums by a single straight line?

A 
triangle

B 
rectangle

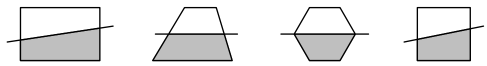
C 
trapezium

D 
regular hexagon

E 
square

SOLUTION **A**

The diagrams on the right show how a rectangle, a trapezium, a regular hexagon and a square can be divided into two trapeziums by a single straight line.



However, when a single straight line cuts through a triangle, the triangle is divided into a triangle and a quadrilateral or, if the line passes through a vertex of the original triangle, two triangles. Hence the shape that cannot be divided into two trapeziums is a triangle.

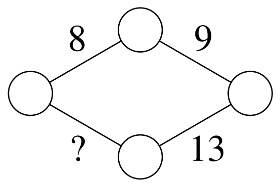
2. What is the sum of the largest three-digit multiple of 4 and the smallest four-digit multiple of 3?

A 1996 B 1997 C 1998 D 1999 E 2000

SOLUTION **C**

The largest three-digit multiple of 4 is 996. The smallest four-digit multiple of 3 is 1002. Their sum is $996 + 1002 = 1998$.

3. Werner wants to write a number at each vertex and on each edge of the rhombus shown. He wants the sum of the numbers at the two vertices at the ends of each edge to be equal to the number written on that edge. What number should he write on the edge marked with the question mark?

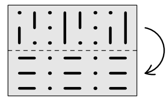


A 11 B 12 C 13 D 14 E 15

SOLUTION **B**

The number on each edge is equal to the sum of the two numbers at the vertices at the ends of the edge. Therefore, the sum of the numbers on two opposite edges will equal the sum of the numbers at all four vertices. Therefore $8 + 13 = 9 + ?$ and hence $? = 12$.

4. Kristina has a piece of transparent paper with some lines marked on it.
She folds it along the central dashed line, as indicated.
What can she now see?



A 2:6:9   B 2:6:6   C 5:6:9   D 2:8:6   E 5:8:9

Solution **C**

Each digit in the completed number contains three horizontal line segments. Since the markings above the fold will appear the other way up when folded, the first digit in the completed number contains a vertical line top left and a vertical line bottom right in addition to the three horizontal lines and so is a 5. The second digit in the completed number contains a double length vertical line on the left and a vertical line bottom right in addition to the three horizontal lines and so is a 6. Similarly, the third digit in the completed number contains a double length vertical line on the right and a vertical line top left in addition to the three horizontal lines and so is a 9. Therefore the number that will be seen is 569.

5. John has 150 coins. When he throws them on the table, 40% of them show heads and 60% of them show tails. How many coins showing tails does he need to turn over to have the same number showing heads as showing tails?

A 10   B 15   C 20   D 25   E 30

Solution **B**

The number of coins that currently show heads is 40% of 150, which is 60. To have the same number of heads showing as tails, John needs 75 coins showing heads. Therefore the number of coins he needs to turn over is $75 - 60 = 15$.

6. Anna has five circular discs, each of a different size. She decides to build a tower using three of her discs so that each disc in her tower is smaller than the disc below it. How many different towers could Anna construct?

A 5   B 6   C 8   D 10   E 15

Solution **D**

Let the discs be numbered 1, 2, 3, 4 and 5 in increasing order of size. The possible towers using three discs that Anna can build are, with the first number on top and then going down the tower, 123, 124, 125, 134, 135, 145, 234, 235, 245 and 345. Therefore she can build 10 different towers.

7. Evita wants to write the numbers 1 to 8 in the boxes of the grid shown, so that the sums of the numbers in the boxes in each row are equal and the sums of the numbers in the boxes in each column are equal. She has already written numbers 3, 4 and 8, as shown. What number should she write in the shaded box?

	4		(shaded)
3		8	

A 1 B 2 C 5 D 6 E 7

Solution **E**

Let the unknown numbers in the boxes be p, q, r, s and t, as shown. The sum of the numbers 1 to 8 is 36 and hence the sum of the numbers in each row is $36 \div 2 = 18$ and the sum of the numbers in each column is $36 \div 4 = 9$. Therefore $p = 9 - 3 = 6$, $r = 9 - 8 = 1$ and hence $s = 18 - 6 - 4 - 1 = 7$. Therefore Evita should write the number 7 in the shaded box.

p	4	r	s
3	q	8	t

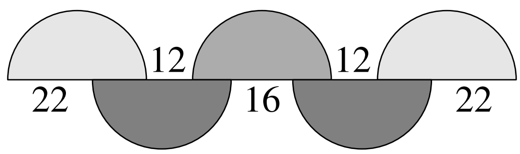
8. Theodorika wrote down four consecutive positive integers in order. She used symbols instead of digits. She wrote the first three integers as □◇◇, ♡△△, ♡△□. What would she write in place of the next integer in the sequence?

A ♡♡◇ B □♡□ C ♡△◇ D ♡◇□ E ♡△♡

Solution **E**

Since the first symbol changes from the first to the second integer, we know the hundreds digit must have changed. Therefore ◇ = 9, △ = 0 and ♡ = □ + 1. Also, since the last digit of the third integer is □, we know that □ = 1 and hence that ♡ = 2. Therefore the next number would be 202, which she would write as ♡△♡.

9. The diagram shows five equal semicircles and the lengths of some line segments. What is the radius of the semicircles?



A 12 B 16 C 18 D 22 E 28

Solution **C**

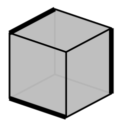
Let the radius of each semicircle be r. From the diagram we can see that $2r + 12 + 2r + 12 + 2r = 22 + 2r + 16 + 2r + 22$ and hence $6r + 24 = 4r + 60$. Therefore $2r + 24 = 60$ and so $2r = 36$ and hence $r = 18$.

10. Some edges of a cube are to be coloured red so that every face of the cube has at least one red edge. What is the smallest possible number of edges that could be coloured red?

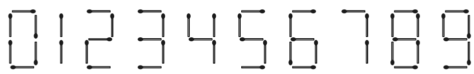
A 2 B 3 C 4 D 5 E 6

Solution **B**

A cube has six faces. An individual edge forms the boundary of only two faces. Therefore at least $6 \div 2 = 3$ edges must be coloured red. It is possible, as shown, to choose three edges which between them include an edge of each face. Hence the smallest number of edges that could be coloured red is 3.



11. Matchsticks can be used to write digits, as shown in the diagram. How many different positive integers can be written using exactly six matchsticks in this way?



A 2 B 4 C 6 D 8 E 9

Solution **C**

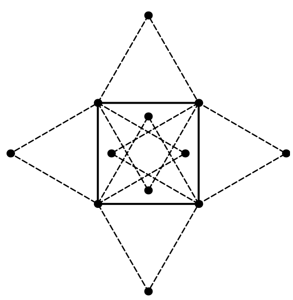
The number of matchsticks used for each of the digits 0 to 9 is 6, 2, 5, 5, 4, 5, 6, 3, 7 and 6 respectively. Hence, the positive integers that can be made using exactly six matches are 6, 9, 14, 41, 77 and 111. Therefore 6 different positive integers can be made.

12. A square with side-length 10 cm long is drawn on a piece of paper. How many points on the paper are exactly 10 cm away from two of the vertices of this square?

A 4 B 6 C 8 D 10 E 12

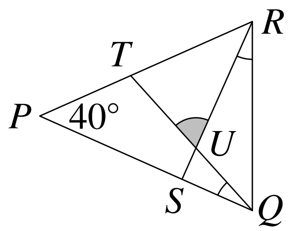
Solution **E**

For each of the four pairs of adjacent vertices, there is one point outside the square and one point inside the square that is exactly 10 cm from each vertex in the pair. Also each of the four vertices themselves is exactly 10 cm from the two adjacent vertices. Hence, there are 12 points in total that are exactly 10 cm from two of the vertices of the square and these points are all distinct. The diagram below shows the positions of these 12 points.



13. In the diagram shown, sides PQ and PR are equal. Also $\angle QPR = 40°$ and $\angle TQP = \angle SRQ$. What is the size of $\angle TUR$?

A 55° B 60° C 65° D 70° E 75°

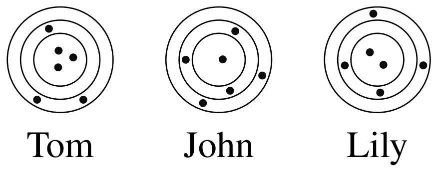


Solution **D**

Let $\angle SRQ$ be $x°$. Since sides PR and PQ are equal, triangle PQR is isosceles and hence $\angle PRQ = \angle PQR = (180° - 40°)/2 = 70°$. Therefore, since we are given that $\angle TQP = \angle SRQ$, we have $\angle UQR$ is $70° - x°$. Hence, since the exterior angle of a triangle is equal to the sum of the interior opposite angles, $\angle TUR$ is $70° - x° + x° = 70°$.

14. Tom, John and Lily each shot six arrows at a target. Arrows hitting anywhere within the same ring scored the same number of points. Tom scored 46 points and John scored 34 points, as shown. How many points did Lily score?

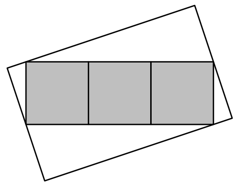
A 37 B 38 C 39 D 40 E 41



Solution **D**

Tom hit the inner ring three times, the middle ring once and the outer ring twice and scored 46 points. John hit the inner ring once, the middle ring three times and the outer ring twice and scored 34 points. Together they scored 80 points from hitting each ring four times. Lily hit each ring twice and so scored half of the total score of Tom and John. Therefore Lily scored 40 points.

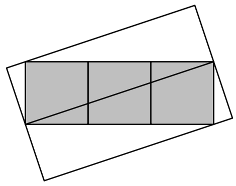
15. The diagram shows a smaller rectangle made from three squares, each of area 25 cm^2, inside a larger rectangle. Two of the vertices of the smaller rectangle lie on the mid-points of the shorter sides of the larger rectangle. The other two vertices of the smaller rectangle lie on the other two sides of the larger rectangle. What is the area, in cm^2, of the larger rectangle?



A 125 B 136 C 149 D 150 E 172

SOLUTION **D**

Join the vertices of the smaller rectangle that lie on the shorter sides of the larger rectangle, as shown in the diagram. This line splits the smaller rectangle into two congruent triangles, each with base equal to the length of the longer sides of the larger rectangle and with perpendicular height equal to half the length of the shorter sides of the larger rectangle.



Therefore the area of each triangle is equal to half the area of the smaller rectangle. Since each triangle is half of the three shaded squares, the area of each triangle, in cm^2, is $(3 \times 25) \div 2 = 37.5$. Hence the area of the larger rectangle, in cm^2, is $4 \times 37.5 = 150$.

16. The sum of 2023 consecutive integers is 2023. What is the sum of digits of the largest of these integers?

A 4 B 5 C 6 D 7 E 8

SOLUTION **A**

Let the middle integer of the list be n. Since the 2023 digits are consecutive, the complete list is $n - 1011$, $n - 1010$, ..., $n - 1$, n, $n + 1$, ..., $n + 1010$, $n + 1011$ with sum $2023n$. Therefore $2023n = 2023$ and so $n = 1$. Hence the largest integer of the list is $1 + 1011 = 1012$, which has digit sum 4.

17. Some beavers and some kangaroos are standing in a circle. There are three beavers in total and no beaver is standing next to another beaver. Exactly three kangaroos stand next to another kangaroo. What is the number of kangaroos in the circle?

A 4 B 5 C 6 D 7 E 8

Solution **B**

Any two beavers are separated by one or more kangaroos. If there were two, or more, separate groups of two, or more, kangaroos standing next to another kangaroo, there would be four, or more, kangaroos standing next to each other. Since the question tells us that there are exactly three kangaroos who stand next to another kangaroo, they are in a group of three and there are no more kangaroos standing next to another kangaroo.

Therefore, since there are three beavers in the circle and none of the beavers is next to another beaver, the arrangement of animals round the circle, starting with one of the beavers, is B K B K B KKK and hence there are exactly five kangaroos in the circle.

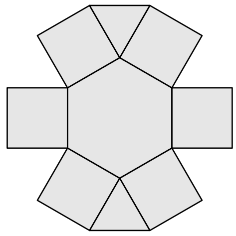
18. Snow White organised a chess competition for the seven dwarves, in which each dwarf played one game with every other dwarf. On Monday, Grumpy played 1 game, Sneezy played 2, Sleepy 3, Bashful 4, Happy 5 and Doc played 6 games. How many games did Dopey play on Monday?

A 1 B 2 C 3 D 4 E 5

Solution **C**

All the following arguments refer to the games played on Monday. Doc played 6 games and so played all the other dwarves. Therefore, Grumpy's game was against Doc. Similarly, since Happy played 5 games and didn't play against Grumpy and Sneezy only played two, Sneezy's games were against Happy and Doc. Bashful played 4 games and didn't play against Grumpy or Sneezy and Sleepy only played three games and so Sleepy's games were against Doc, Happy and Bashful. Therefore Dopey played against Doc, Happy and Bashful so played 3 games.

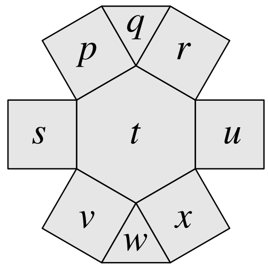
19. Elizabetta wants to write the integers 1 to 9 in the regions of the shape shown, with one integer in each region. She wants the product of the integers in any two regions that have a common edge to be not more than 15. In how many ways can she do this?



A 8 B 12 C 16 D 24 E 32

Solution **C**

Let the integers in the different regions be p, q, r, s, t, u, v, w and x, as shown. First consider the positions of integers 8 and 9. Since the product of integers in two regions with a common edge cannot exceed 15, integers 8 and 9 can only be placed in regions with one edge in common with another region. Therefore 8 and 9 can only be written in the regions marked with s and u in either order and 1 can only be written in the region marked t.



Next consider the position of the integer 7. This can only be in a region with a common edge to the regions with 1 and 2 in. Hence, 7 can only be placed in one of the regions marked p, r, v or x and then must have a 2 in the adjacent triangle. Since the integer 6 can also only be in a region with a common edge to regions containing either 1 or 2, once the position of 7 is decided, the position of 6 is fixed as being in the square on the opposite side of the triangle to the square containing 7.

Finally, the three remaining regions must be filled with integers 3, 4 and 5 with the 3 being written in the second triangle and the other two integers being written either way round. Therefore the diagram can be filled in $2 \times 4 \times 2 = 16$ ways.

20. There were twice as many children as adults sitting round a table. The age of each person at the table was a positive integer greater than 1. The sum of the ages of the adults was 156. The mean age of the children was 80% less than the mean age of the whole group.
What the sum of the ages of the children?

A 10 B 12 C 18 D 24 E 27

Solution **D**

Let the number of adults be n and let the total of the ages of the children be T. The information in the question tells us that the mean age of the children is 20% or $\frac{1}{5}$ of the mean age of the whole group.

Therefore $\frac{1}{5} \times \frac{156+T}{3n} = \frac{T}{2n}$. Therefore $2(156 + T) = 3 \times 5T$ and hence $312 = 13C$. This has solution $T = 24$.

21. Martin is standing in a queue. The number of people in the queue is a multiple of 3. He notices that he has as many people in front of him as behind him. He sees two friends, both standing behind him in the queue, one in 19th place and the other in 28th place. In which position in the queue is Martin?

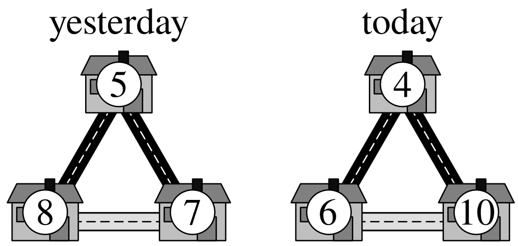
A 14 B 15 C 16 D 17 E 18

Solution **D**

Let the number of people in front of Martin in the queue be N. Therefore, he is in place $N + 1$ in the queue and since his friend in 19th place is behind him we have $N + 1 < 19$ and hence $N \leq 17$. Since there are also N people behind him in the queue, the total number of people in the queue is $2N + 1$ and since his other friend is in 28th place, we have $2N + 1 \geq 28$ and hence $N \geq 14$ as N is an integer.

When we combine these two results for N, we find that the only possible values of N are 14, 15, 16 and 17 with corresponding values for the total number of people in the queue of 29, 31, 33 and 35. Of these, the only total that is a multiple of 3 is 33 and hence $N = 16$ and Martin is in 17th position in the queue.

22. Some mice live in three neighbouring houses. Last night, every mouse left its house and moved to one of the other two houses, always taking the shortest route. The numbers in the diagram show the number of mice per house, yesterday and today. How many mice used the path at the bottom of the diagram ?



A 9 B 11 C 12 D 16 E 19

Solution **B**

Let the number of mice that travelled from the left-hand house to the right-hand house along the bottom path be x and the number of mice that travelled from the right-hand house to the left-hand house be y. Therefore, since every mouse changed house last night, $8 - x$ mice travelled from the left-hand house to the top house and $7 - y$ mice travelled from the right-hand house to the top house. Hence, since there were then 4 mice in the top house, we have $8 - x + 7 - y = 4$ and so $x + y = 11$. Therefore the number of mice that travelled along the bottom path in the diagram is 11.

23. Bart wrote the number 1015 as a sum of numbers using only the digit 7. He used a 7 a total of 10 times, including using the number 77 three times, as shown. Now he wants to write the number 2023 as a sum of numbers using only the digit 7, using a 7 a total of 19 times. How many times will the number 77 occur in the sum?

$$\begin{array}{r} 777 \\ 77 \\ 77 \\ 77 \\ +\quad 7 \\ \hline 1015 \end{array}$$

A 2 B 3 C 4 D 5 E 6

Solution **E**

First note that $777 \times 3 = 2331 > 2023$ and hence the maximum number of times the number 777 could appear in the sum is 2. If there were no 777s in the sum, the maximum value that could be obtained using the digit 7 exactly 19 times is $9 \times 77 + 7 = 700$, which is too small.

Similarly, if there was only one 777 in the sum, the maximum value that could be obtained using the digit 7 exactly 19 times is $777 + 8 \times 77 = 1393$, which is also too small. Therefore the number 777 is required twice. Now note that $2023 - 2 \times 777 = 469$ and that $469 = 6 \times 77 + 7$. Therefore the sum Bart should write would consist of 777 twice, 77 six times and a single 7, which does consist of the digit 7 a total of 19 times.

24. Jake wrote six consecutive numbers on six white pieces of paper, one number on each piece. He stuck these bits of paper onto the top and bottom of three coins. Then he tossed these three coins three times. On the first toss, he saw the numbers 6, 7 and 8 and then coloured them red. On the second toss, the sum of the numbers he saw was 23 and on the third toss the sum was 17. What was the sum of the numbers on the remaining three white pieces of paper?

A 18 B 19 C 23 D 24 E 30

Solution **A**

The sum of the three numbers visible on the first toss was $6 + 7 + 8 = 21$. Therefore, since the sum of the numbers visible on the second toss was 23, which is greater than 21, and the sum of the numbers visible on the third toss was 17, which is less than 21, Jake's list of consecutive numbers included at least one number greater than 8 and at least one number less than 6.

Hence the two possibilities for his six consecutive numbers are 4, 5, 6, 7, 8, 9 and 5, 6, 7, 8, 9, 10. However, for the second list, the minimum total of three different numbers is $5 + 6 + 7 = 18$ which is more than the observed third total. Therefore the six numbers Jake used were 4, 5, 6, 7, 8 and 9 and hence the sum of the three numbers on the three remaining white pieces of paper was $4 + 5 + 9 = 18$.

25. A rugby team scored 24 points, 17 points and 25 points in the seventh, eighth and ninth games of their season. Their mean points-per-game was higher after 9 games than it was after their first 6 games. What is the smallest number of points that they could score in their 10th game for their mean number of points-per-game to exceed 22?

A 22 B 23 C 24 D 25 E 26

Solution $\boxed{\textbf{C}}$

Let X be the total number of points scored after 6 games and let y be the number of points scored in the 10th game. Since the mean score after 9 games is greater than the mean score after 6 games, we have $\frac{X}{6} < \frac{X+24+17+25}{9}$ and hence $3X < 6 \times 66$ and so $X < 132$.

Similarly, since the mean after 10 games is greater than 22, $\frac{X+66+y}{10} > 22$ and hence $X + 66 + y > 220$ and so $X + y > 154$.

Therefore $X \leq 131$ and $X + y \geq 155$. Hence the smallest number of points they could score in the 10th game is $155 - 131 = 24$.